Fast statistical reconstruction algorithm for a CT scanner with flying focal spot

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***Abstract*—This paper is related to the originally formulated 3D statistical model-based iterative reconstruction algorithm for tomography with flying focal spot. The conception proposed here is based on a continuous-to-continuous data model, and the reconstruction problem is formulated as a shift invariant system. The proposed approach resembles the FDK method, significantly improving the quality of the reconstructed images in comparison to that FBP referential algorithm, so allowing a reduction in the x-ray dose absorbed by a patient. The presented here approach avoids problems involved with nutating approaches usually applied in CT scanners with dual source x-ray tubes. Computer simulations have shown that the reconstruction method presented here outperforms standard FDK methods with regard to the image quality obtained and can be competitive in terms of time of calculation.**

***Index Terms*—dual source CT, flying focal spot, iterative reconstruction algorithm.**

I. Introduction

MANY years after the ivnention of the CT, new tomograph constructions are constantly being sought. In the beginning of 2004 medical spiral scanners were introduced that makes use of a flying focal spot (see e.g. [1]). The flying focal spot in the α-direction aims at inreasing the sampling density of the integral lines in reconstruction planes, and in the z-direction at increasing the sampling density just in z-direction. In practice, several approaches to reconstruction problem were formulated which allow for utilization of projections obtained from scanners with the flying focal spot. One of the most important among them is the adaptive multiple plane reconstruction (AMPR) algorithm, representing the class of nutating reconstruction method (see e.g. [2]). The AMPR conception is an extention of the advanced single slice rebinning (ASSR) method (see e.g. [1]). Despite the fact that the AMPR method is used commercially this rebinning approach has several serious drawbacks, mainly involved with some approximations and corrections needed to perform during realization of reconstruction procedure. Because of the linear nature of the operations carried out in the AMPR algorithm the ability to suppress noise in this approach is limited. It this context, an important circumstance is that recently the most significant problem in medical CT has been the development of image reconstruction methods which would enable the reduction of the impact of measurement noise on the quality of tomography images and thus decrease the dose of X-ray radiation absorbed by patients during examinations. Some of the most interesting research directions in this area are statistical reconstruction methods, especially those belonging to the MBIR (Model-Based Iterative Reconstruction) approach [3], where a probabilistic model of the measurement signals is taken into account. Unfortunately, those methods have some very serious drawbacks from the theoretical and practical point of view: for instance, if the image resolution is set to be $\mathrm{I} \times \mathrm{I}$ pixels, the calculation complexity of the problem is proportional to I^4, the statistical reconstruction procedure based on this methodology necessitates simultaneous calculations for all the voxels in the range of the reconstructed 3D image, the size of the forward model matrix $\mathbf{A}$ is huge, and this makes it often necessary to calculate them in every iteration of the reconstruction algorithm. In this case, the reconstruction problem is extremely ill-conditioned, and it is necessary to introduce an *a priori* term (often referred to in the literature as a regularization term) into the objective, and this leads to the use of the MAP model. The problems connected with the use of a methodology based on the D-D data model can be reduced by using a strategy of reconstructed image processing based on a continuous-to-continuous (C-C) data model. In previous papers we have shown how to formulate reconstruction problems consistent with the C-C mode and with the ML methodology for parallel scanner geometry [4], and finally for the spiral cone-beam scanner [5]. However, an approach to the reformulation of the reconstruction problem from parallel to real scanner geometries, called rebinning, was applied there. Much more convenient are FDK-type algorithms, and in this paper, we present a conception of the direct use of spiral cone-beam projections to a statistical reconstruction algorithm for dual source CT scanner.

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II. Reconstruction Algorithm

Our reconstruction method is based on the well-known maximum-likelihood (ML) estimation [7]. We propose here an optimization formula which is consistent with the C-C data model, in the following form:

$$\mu_{\min} = \arg\min_{\mu} \left(\sum_{i=1}^{\mathrm{I}} \sum_{j=1}^{\mathrm{I}} \left(\Delta_s^2 \sum_{\bar{i}=1}^{\mathrm{I}} \sum_{\bar{j}=1}^{\mathrm{I}} \mu^* \left(x_{\bar{i}}, y_{\bar{j}} \right) \cdot h_{\Delta i, \Delta j} - \tilde{\mu} \left(x_i, y_j \right) \right)^2 \right), \quad (1)$$

where I is a resolution of the reconstructed image, coefficients $h_{\Delta i, \Delta j}$ are precalculated according to the following relation:

$$h_{\Delta i, \Delta j} = \Delta_\alpha \sum_{\psi=0}^{\Psi-1} int \left(\Delta i \cos \psi \Delta_\alpha + \Delta j \sin \psi \Delta_\alpha \right), \quad (2)$$

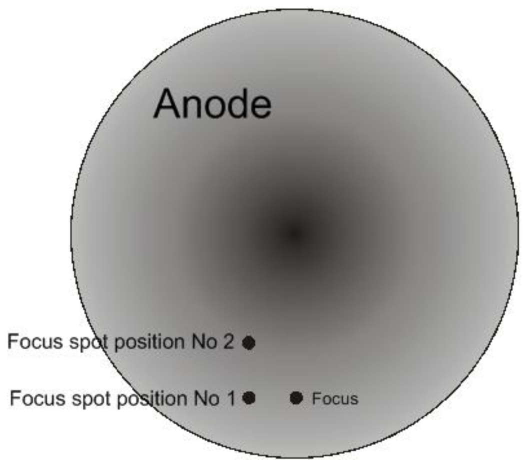

Fig. 1. The movement of focal spot with flying spot technique used in our experiments.

and $\tilde{\mu}(i,j)$ is an image obtained by way of a back-projection operation; Δ_s is a resolution of projections in a hypothetical parallel geometry of scanner; $int(\Delta s)$ is an interpolation function used in the back-projection operation; every projection is carried out after a rotation by Δ_α.

The presence of a shift-invariant system in the optimization problem (1) implies that this system is much better conditioned than the least squares problem present in the D-D approach [6].

In our considerations, we took into account the dual source CT scanner with two flying spot positions, as shown in Fig. 1. The focus spot position No. 1 is involved only with movement in α-direction, and the focus spot position No. 2 corresponds to the focal movement both in α-direction and z direction.

The conception presented here is a full 3D iterative reconstruction algorithm for spiral cone-beam scanner geometry. This algorithm is based on the one of the principal reconstruction methods devised for the cone-beam spiral scanner, i.e. the generalized FDK algorithm. The statistical reconstruction method proposed by us consists of two steps, namely: a back-projection operation described by the following relations:

$$\breve{\mu}^{'}(x,y) \approxeq \frac{1}{2}\int_{0}^{2\pi}\int_{-\beta_m}^{\beta_m} \frac{R_{fd}}{\sqrt{{R_{fd}}^2+z_k^2}} \cdot p^h\left(\beta,{\alpha^{'}}^h,z_k\right) int_L(\Delta\beta)\, d\beta d{\alpha^{'}}^h, \tag{3}$$

for the first focal spot position, where R_{fd} is the nominal focus-detector distance, ${\alpha^{'}}^h=\alpha^h+\delta_{\alpha^{'}}$; $\delta_{\alpha^{'}}$ is the focal spot deviation in angular direction for the first focal spot position, and

$$\breve{\mu}^{''}(x,y) \approxeq \frac{1}{2}\int_{0}^{2\pi}\int_{-\beta_m}^{\beta_m} \frac{R_{fd}^{''}}{\sqrt{{R_{fd}^{''}}^2+z_k^2}} \cdot p^h\left(\beta,{\alpha^{''}}^h,z_k\right) int_L(\Delta\beta)\, d\beta d{\alpha^{''}}^h, \tag{4}$$

for the second focal spot position, where $R_{fd}^{''}=R_{fd}+\delta_{R_f}$; δ_{R_f} is the focal spot deviation in radial direction, ${\alpha^{''}}^h=\alpha^h+\delta_\alpha$; δ_α is the focal spot deviation in angular direction for the second focal spot position.

After the back-projection operation, an iterative reconstruction procedure according to formula (1) is performed using the image being a sum of two above described images, namely $\breve{\mu}^{'}(x,y)$ and $\breve{\mu}^{'}(x,y)$, i.e.:

$$\breve{\mu}(x,y)=\breve{\mu}^{'}(x,y)+\breve{\mu}^{''}(x,y). \tag{5}$$

Generally, it could be used any number of the focal spot positions, which could be moved both in z and in angular α directions.

It is worth noting that an interpolation function which is taken into account in relation (2) has the following form:

$$int_L(\Delta s)=\begin{cases} \frac{1}{\Delta_s}\left(1-\frac{|\Delta s|}{\Delta_s}\right) & \text{for} \quad |\Delta s|\leq\Delta_s \\ 0 & \text{for} \quad |\Delta_s|\geq\Delta_s \end{cases}, \tag{6}$$

where the optimal Δs can be determined using the following equation:

$$\Delta s=R_f\sin\Delta_\beta, \tag{7}$$

where R_f is the focus-isocenter distance.

There are some technical problems with the performing of the interpolation needed during the back-projection operation regarding the second focal spot position. A distribution of integral lines are no more equiangular for this position and the interpolation is performesd at the different resolution in every case. For details regarding method for determining parameters of geometry of scanner with the flying focal spot see e.g. [8] and [9].

Figure 1 depicts this algorithm after discretization and implementation of FFT which significantly accelerates the calculations.

III. Experimental results

In our experiments, we have used projections obtained from a Somatom Definition AS+ (helical mode) scanner with the following parameters: reference tube potential 120kVp and quality reference effective 200mAs, $R_{fd}=1085.6mm$, $\mathrm{R}_f=595\mathrm{mm}$, number of views per rotation $\Psi=1152$, number of pixels in detector panel 736, detector dimensions were $1.09\mathrm{mm}\times1.28\mathrm{mm}$. During the experiments, the size of the processed image was fixed at 512×512 pixels. A discrete representation of the matrix $h_{\Delta x,\Delta y}$ was established before the reconstruction process was started, and these coefficients were fixed (transformed into the frequency domain) for the whole iterative reconstruction procedure. The image obtained after the back-projection operations was then subjected to a process of reconstruction (optimization) using an iterative procedure. A specially prepared result of an FBP reconstruction algorithm was chosen as the starting point of this procedure (using projections obtained from the first focal spot position. It is worth noting that our reconstruction procedure was performed without any regularization regarding the objective function from (1). The iterative reconstruction procedure was implemented for a computer with 10 cores, i.e. with an Intel i9-7900X BOX/3800MHz processor (the iterative reconstruction

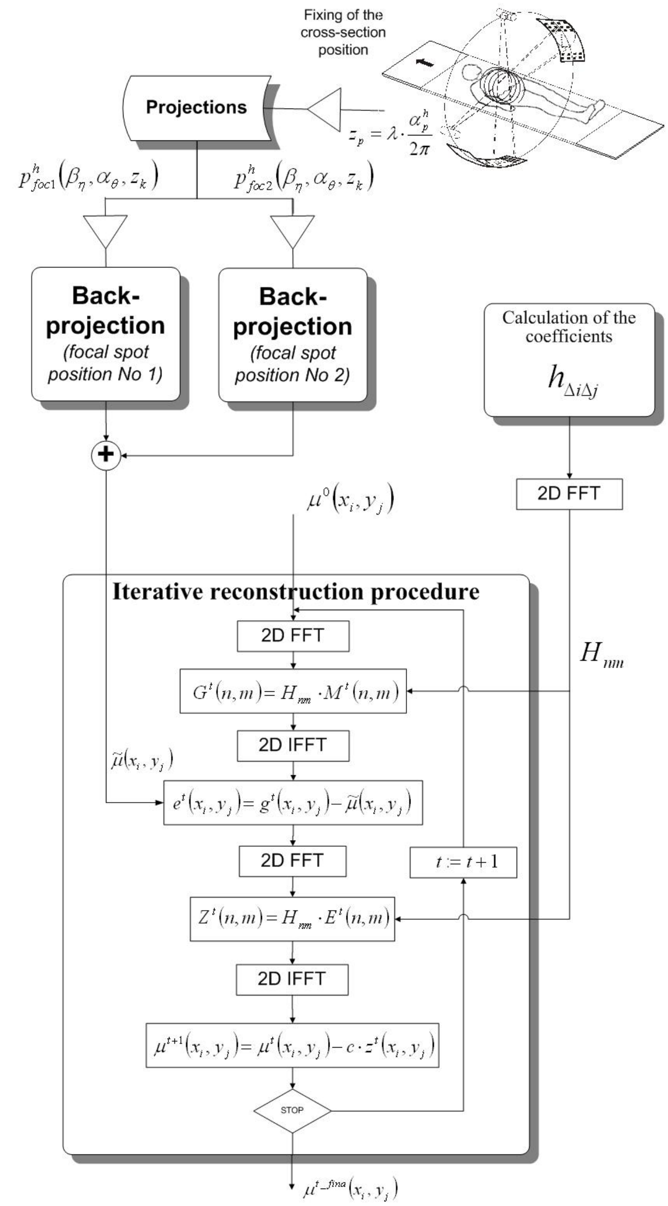

Fig. 2. Statistical reconstruction algorithm for spiral cone-beam scanner with flying focal spot.

procedure was implemented at assembler level), and using a GPU type nVidia Titan V. According to an assessment of the quality of the obtained images by a radiologist, 10000 iterations are enough to provide an acceptable image. The same results were achieved for both hardware implementations after about 9s, for the CPU and GPU implementations. One can compare the results obtained by assessing the views of the reconstructed images in Figure 3, where the full dose projections were used, and in Figure 4 where the quarter-dose projections were considered. In the both cases, Figures (A) depict reconstructed images obtained using the standard FDK algorithm (with linear interpolation function and Shepp-Logan kernel), Figures (B) and (C) present reconstructed images where the statistical approach presented in this paper was used: only measurements performed using the first focal spot position (B), all measurements were used (C).

IV. Conclusion

A statistical iterative reconstruction algorithm which can be used in practice for helical cone-beam scanners with flying focal spot has been shown above. We have conducted computer simulations, which proved that our reconstruction method is very fast, above all thanks to the use of FFT algorithms and efficient programming techniques, and it gives satisfactory results regarding the quality of the obtained images, and perspectivelly at a significantly reduced dose of x-rays absorbed by the patient. This algorithm is very easy to implement and open to use multisource technique, regardless the kind of the focal spot movement (both z and angle flying). If the image resolution is assumed to be I $\times$ I pixels, the complexity of the approach implemented here is proportional to $I^2 \log_2 I$. One can note that the iterative reconstruction procedure was performed without introducing any additional regularization term, using only an early stopping regularization strategy. It should be underlined that the price of the hardware used is relatively low (about 5000 USD in both cases).

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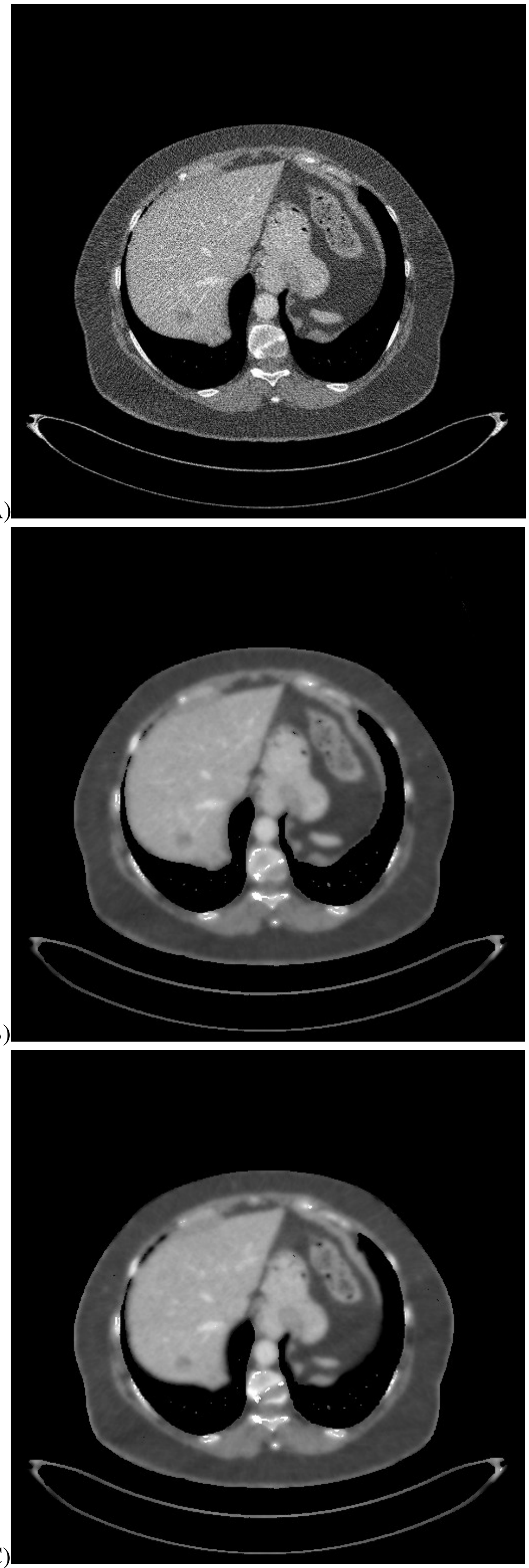
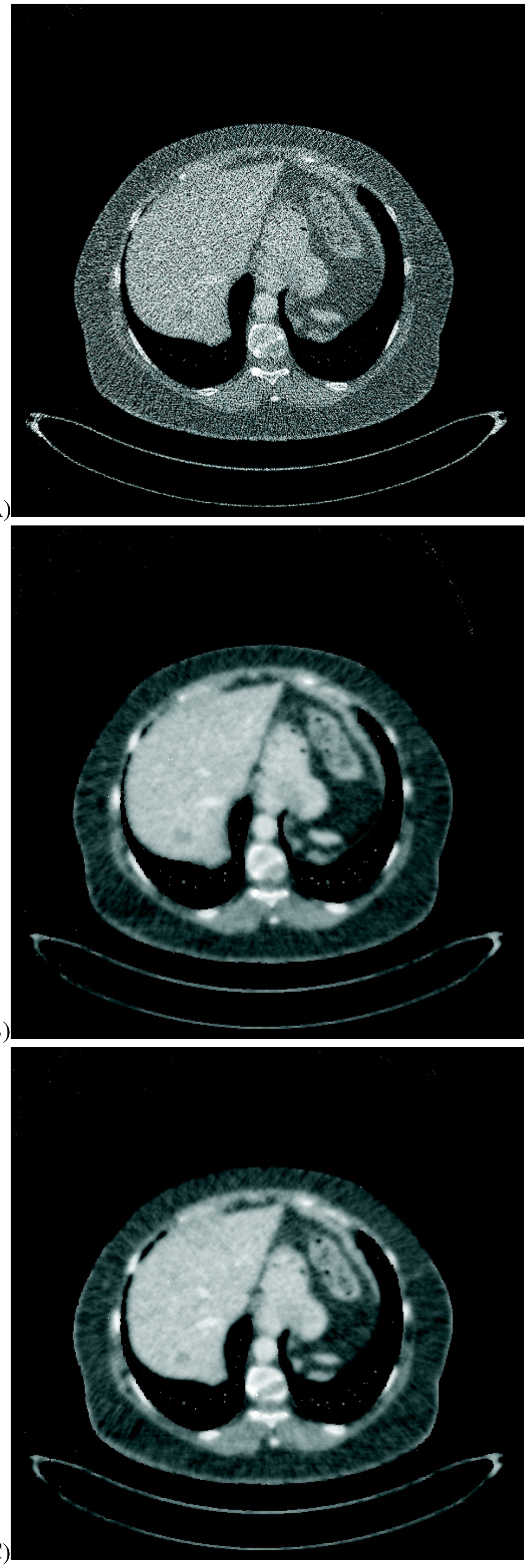


A)
B)
C)

Fig. 3. View of the reconstructed image (a case with pathological changes in the liver) using full-dose projections with application of: the standard FDK algorithm (A); the statistical method presented in this paper (the first focal spot position) (B); the statistical method presented in this paper (both focal spot positions) (C).



A)
B)
C)

Fig. 4. View of the reconstructed image (a case with pathological changes in the liver) using quarter-dose projections with application of: the standard FDK algorithm (A); the statistical method presented in this paper (the first focal spot position) (B); the statistical method presented in this paper (both focal spot positions) (C).