Realizations of the Statistical Reconstruction Method Based on the Continuous-to-Continuous Data Model

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Abstract. The presented paper describes a successfully parallel implementation of the statistical reconstruction method based on the continuous-to-continuous model using both CPU and GPU hardware approaches. Data were obtained from a commercial computer tomography device which were saved in DICOM standard file. The implemented reconstruction algorithm is formulated taking in two consideration the statistical properties of signals obtained by x-ray CT and the continuous-to-continuous data model. During our experiments, we tested the speed of the implemented algorithm and we optimized it in terms of the critical parameter which is very important regarding the potential use of this solution in clinical practice.

Keywords: Image reconstruction from projections · Statistical iterative reconstruction algorithm · Computer tomography

1 Introduction

The main challenge in x-ray computed tomography is to improve the resolution of reconstructed images and/or decrease the x-ray dose absorptions by a patient during examination, while maintaining the quality of the CT images obtained, and therefore this is a barrier to the development of this wildly-spread medical imaging technique. It is why the statistical reconstruction methods are being so intensively developed. In the statistical approach, signal processing is adaptive to the statistic of measurements present in a given image technique and in this way, we can reduce the dose absorbed by a patient during an examination (see e.g. in [1,2]). This paper presents our investigations on this challenge. We present hire results obtained using implementations of statistical reconstruction algorithm based on a continuous-to-continuous data model. Because the time is a crucial parameter in medical practice, it is very important to perform a whole reconstruction procedure in a limited time, i.e. within a few seconds. To make this

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time as short as possible we strived to implement as parallel realizations of this algorithm using a multi-thread assembler working on AVX512 vector registers and a few different GPUs accelerators.

2 Statistical Reconstruction Algorithm

Our reconstruction method is based on the well-know maximum-likelihood (ML) estimation [3–5]. In most cases, the objective in those solutions is devised according to a discrete-to-discrete (D-D) data model. However, this scheme has some very serious drawbacks, namely: the statistical reconstruction procedure based on this methodology necessitates simultaneous calculations for all the voxels, in the range of the reconstructed 3D image, the size of the forward model matrix **A** is huge, and this makes it often necessary to calculate them in every iteration of the reconstruction algorithm. In this case, the reconstruction problem is extremely ill-conditioned, and it is necessary to introduce an *a priori* term (often referred to in the literature as a regularization term) into the objective, and this leads to the use of the MAP model. We propose here an optimization formula which is consistent with the C-C data model, in the following form:

$$\mu_{\min} = \arg\min_{\mu} \left(\int_{x} \int_{y} \left(\int_{\bar{x}} \int_{\bar{y}} \mu\left(\bar{x},\bar{y}\right) \cdot h_{\Delta x,\Delta y} d\bar{x} d\bar{y} - \tilde{\mu}\left(x,y\right) \right)^{2} dxdy \right), \quad (1)$$

where $\tilde{\mu}(x,y)$ is an image obtained by way of a back-projection operation, obtained theoretically in the following way:

$$\tilde{\mu}\left(x,y\right) \approxeq \int_{0}^{2\pi} \int_{-\beta_{max}}^{\beta_{max}} p^{h}\left(\beta,\alpha^{h},z_{k}\right) \frac{R_{fd}}{\sqrt{R_{fd}^{2}+z_{k}^{2}}} int_{L}\left(\Delta\beta\right) d\beta d\alpha, \quad (2)$$

where $p^h(\beta, \alpha^h_\psi, z_k)$ are measurements carried out using a spiral cone-beam scanner, R_{fd} is the SDD (Source-to-Detector Distance), and the coefficients $h_{\Delta i,\Delta j}$ can be pre-calculated according to the following relation:

$$h_{\Delta x,\Delta y} = \int_{0}^{2\pi} int\left(\Delta x \cos\alpha + \Delta y \sin\alpha\right) d\alpha, \quad (3)$$

and *int(*Δs*)* is an interpolation function.

The presence of a shift-invariant system in the optimization problem implies that this system is much better conditioned than the least squares problems present in the referential approach [6]. It is necessary for a computer implementation of the above C-C model to discretization Eqs. (1)–(3). After the process of discretization Eqs. (1)–(3) can be presented in the following form (4)–(6)

$$\mu_{\min} = \arg\min_{\mu} \left(\Delta_{xy}^{2} \sum_{i} \sum_{j} \left(\Delta_{\bar{x}\bar{y}}^{2} \sum_{\bar{i}} \sum_{\bar{j}} \mu(\bar{x_i},\bar{y_j}) h_{\Delta i,\Delta j} - \tilde{\mu}(x_i,y_j) \right) \right) \quad (4)$$

where: $\Delta_{x,y}$, is a distance between pixels in a reconstruction image; $\Delta_i = |\bar{i} - i|$, $\Delta_j = |\bar{j} - j|$, are distances between pixels in the reconstruction images in the x and y directions, respectively; $\tilde{\mu}(x_i, y_j)$ is a discrete form of an image obtained by the way of back-projection operation, obtained in the following way:

$$\tilde{\mu}(x_i, y_j) \approxeq \Delta_\alpha \Delta_\beta \sum_\psi \sum_l p^h(\beta_l, \alpha_\psi^h, z_k) \frac{R_{fd}}{\sqrt{R_{fd}^2 + z_k^2}} int_{lin}(\Delta\beta) \tag{5}$$

where $p^h(\beta_l, \alpha_\psi^h, z_k)$ are measurements carried out using a real spiral cone-beam scanner (see e.g. [7]); R_{fd} is the SDD (Source-to-Detector Distance); Δ_α is an angular raster between angles of projections; Δ_β is an angular distance between the radiations detectors in columns of the detector area; $int_{lin}\Delta\beta$ is a linear interpolation function; (k, l) indicates a positions of a given detector in the detector area; and the coefficients $h_{\Delta i, \Delta j}$ can be pre-calculated according to the following relations:

$$h_{\Delta i, \Delta j} = \Delta_\alpha \sum_\psi int(\Delta_i \cos\psi * \Delta_\alpha + \Delta_j \sin\psi * \Delta_\alpha) \tag{6}$$

The most important thing in this approach is the possibility of an implementation of the fast fourier transform (FFT) algorithm to solve optimization problem (4) in an iterative procedure. The main aim of our paper is to present the acceleration of the performers of the calculations regarding this application of FFT using its parallel realizations (see e.g. [8]).

The conception of full 3D reconstructions algorithm proposed in this paper is based on one of the principal reconstruction methods devised for the cone-beam spiral scanner, i.e. the generalized FDK approach to the reconstruction problem.

The statistical reconstruction algorithm formulated by us consists of two steps, namely: a back-projection operation represented by relation 5 and an iterative reconstruction procedure described to formula 4. The whole statistical reconstruction algorithm, including of implementation of the FFT, proposed by us is the depicted in Fig. 1.

3 Results of Reconstruction Images

In Fig. 2, we present the reconstructed image obtained using the presented here method.

The above showed result demonstrates that the algorithm works properly, and the reconstructed image contains a lot of details while minimizing the noise overhead. Thanks to this solution, it will be possible to see a potential threat to the patient's life or reduce the dose of x-ray radiation in order to obtain the same quality of the diagnostic images in comparison to the existing reconstruction methods.

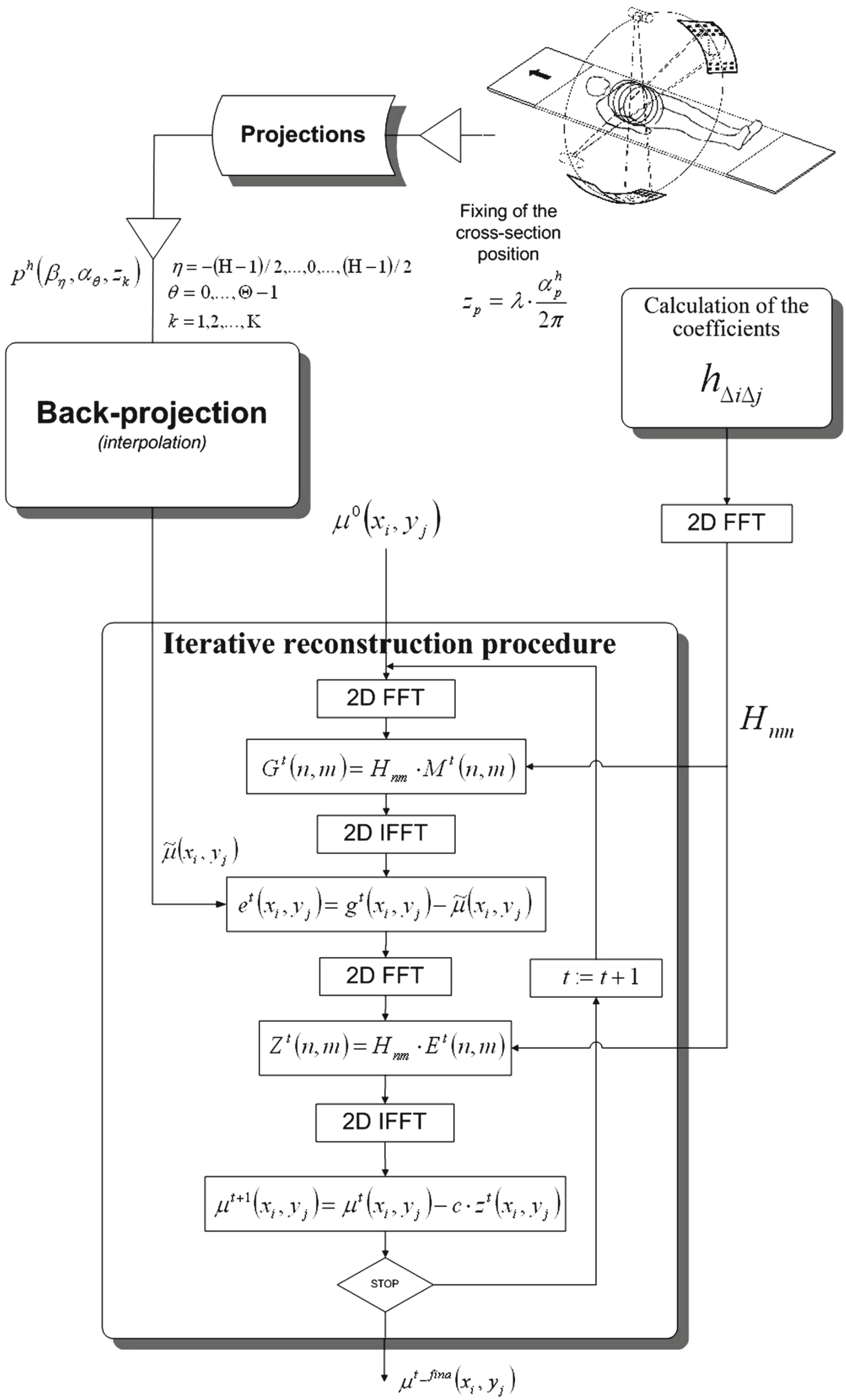

Fig. 1. An image reconstruction algorithm for the cone-beam computer tomograph

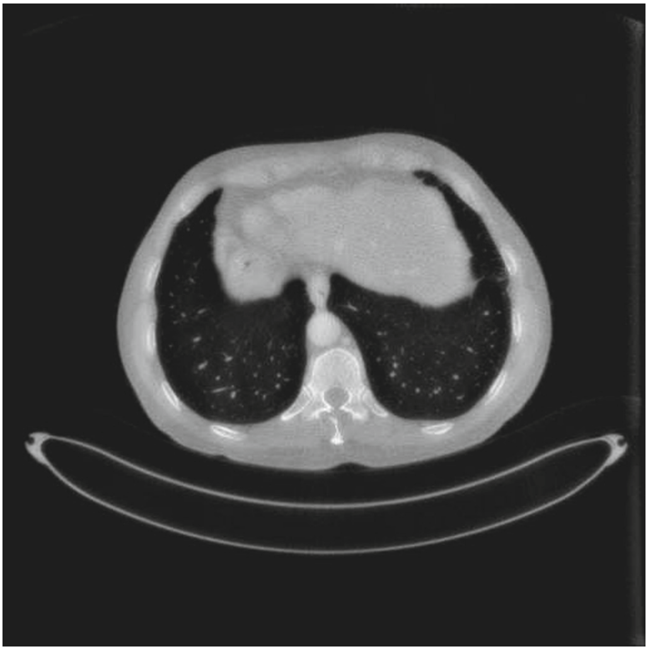

Fig. 2. Reconstructed images from the cone-beam computer tomograph with spiral pitch 0.6 after 20000 iterations.

4 Hardware Description and Time Results

Here we present our results regarding the time of the performance of the reconstruction process which is required to receive the result presented in Sect. 3. Our hardware platform is a computer with processor Intel i9-7900X BOX, mainboard ASUS TUF X299 MARK 2, LGA 2066, X299, with 32 GB RAM DDR4/3200 MHz. This equipment is managed by the operating system: Microsoft Windows 10 professional 64 bit. On this system is running our program which we describe in Sect. 2.

In Table 1, we show time result for the application which is working only on CPU. This version of the application is developed in Assembler which uses special vector registers (AVX 512), which are available only in top models of mainstream Intel's processors, or in servers processors which are very expensive compared to mainstream processors. The discrepancy between standard deviation varies with CPU because the operating system takes some of the resources to maintain the computer.

In Table 2, we show time result for the application which is working only on GPU accelerators. We could compare those accelerators, and draw conclusions that the program is very stable about the time of performance because the deviation is extremely small. Additionally, the application it is very susceptible to parallelization because the time of the one iteration is getting the smaller the more CUDA Cores are assembled in GPU Accelerator.

The situation can be much better in future Graphics Cards which will have more CUDA cores. The border of its is approximate 130 thousand CUDA cores, so it is about 25 square more than today have the best graphics card from Nvidia Company (Titan V).

Table 1. Obtained results regarding the time of the performance of the reconstruction procedure using a multi-threading CPU, i.e. Intel i9-7900X (10-cores, 20-threads). An application created in the assembler programming language with multithreading.

Threads:	4	8	10	16	20
Avg. time 30000 [ms]	63 724,36	33 571,42	29 836,34	30 532,14	27 905,62
Avg. time 20000 [ms]	42 482,91	22 380,95	19 890,89	20 354,76	18 603,75
Avg. time 10000 [ms]	21 241,45	11 190,47	9 945,45	10 177,38	9 301,87
Time/1 iteration [ms]	2,124145	1,119047	0,994545	1,017738	0,930187
HT effectiveness	-	-	-	0,909468	0,935290
Median for 30000	63 694	33 542,5	29 800	30 566	27 854
Deviation std.	135,69	117,32	217,58	193,88	391,76

Table 2. Obtained results regarding the time of the performance of the reconstruction procedure using different models of GPU accelerator. An application created in the CUDA programming language.

GPU	MSI GTX 1050	ASUS GTX 1080 Ti	nVidia Titan V
Avg. time 30000 [ms]	2 562 175,10	49 699,71	28 858,40
Avg. time 20000 [ms]	170 845,28	33 132,52	19 224,48
Avg. time 10000 [ms]	85 467,24	16 593,00	9 616,75
Time/1 iteration [ms]	8,540583	1,656657	0,961947
Median for 30000	256 229,55	49 703,68	28 861,24
Deviation std.	0,160806	0,310476	0,010239

5 Conclusion

We have shown that the statistical approach to the image reconstruction problem based on the continuous-to-continuous data model, which was originally formulated for scanners with parallel beam geometry, can be also utilized in helical CT scanners. Computer simulations have been performed, which prove that our reconstruction method is very fast, mainly thanks to the use of an FFT algorithm. The computational complexity for the proposed reconstruction algorithm is proportional to $8I^2 \log_2(2I)$, wherein it is approximate I^4 operations for the referential approach based on the discrete-to-discrete data model. Thanks to the parallel implementations of the proposed method, the whole reconstruction procedure takes about 7 s regarding a single image, what is absolutely acceptable from the clinical point of view (the obtained images are with satisfactory quality with strongly suppressed noise). It is worth to note that for the referential reconstruction method doctors obtain the first diagnostic CT images after 10–90 min. However, computational intelligence can find their application in reconstruction techniques (see e.g. in [9–19]).

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